A Study of Second Order P – Differential Equation Associated Angular Displacement with A Shaft - IV

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Abstract: This paper is to deliberate the solution of the second order p -differential equation

$$\frac{\partial^2 \psi}{\partial t^2} = \mu^2 \frac{\partial^2 \psi}{\partial x^2}$$

About to a interpellation angular-displacement with shaft - IV. The results are useful in situation appearing in the literature on applied and pure physics mathematics and engineering.

Keywords: $\overline{\mathbf{H}}$ -Function, Generalized Polynomials, Partial Differential Equation, Angular Displacement, Mellin - Barnes Contour Integral.

Introduction: Consider the interpellation of determining the twist ψ (x,t) with a shaft –IV of circular section within x-axis. The displacement ψ (x,t) due primary twist satisfy condition of interpellation , if both ends x = 0, x = λ of shaft are free.

$$\frac{\partial^2 \psi}{\partial t^2} = \mu^2 \frac{\partial^2 \psi}{\partial x^2}, \qquad \ldots(1)$$

where μ is a constant.

$$\frac{\partial \psi}{\partial x}(0,t) = 0, \ \frac{\partial \psi}{\partial x}(x,0) \text{ and } \psi(x,0) = f(x), \qquad \ldots(2)$$

Let

$$f(x) = \left(\sin\frac{\pi x}{2\lambda}\right)^{2\sigma-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_1},\ldots,y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_s}\right]$$

$$.\overline{H}_{P,Q}^{M,N}\left[z\left(\tan\frac{\pi x}{2\lambda}\right)^{2h}\right], \qquad \ldots (3)$$

The following polynomials introduced by Srivastava [12]:

$$S_{N_1,\ldots,N_s}^{M_1,\ldots,m_s}[x_1,\ldots,x_s] = \sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!}\cdots\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}$$

$$B[N_1,\alpha_1;\ldots;N_s,\alpha_s]\, x_1^{\alpha_1}\ldots x_s^{\alpha_s}, \qquad \ldots (4)$$

where $N_{i'} = 0,1,2,\ldots \forall i' = (1,\ldots,s), M_1,\ldots,M_s$ positive (arbitrary) integers and coefficient $[N_1,\alpha_1;\ldots;N_s,\alpha_s]$ are constants (arbitrary), real/ complex.

The following $\overline{H}$-function introduced by Inayat Hussain ([8],[9]) as

$$\overline{H}_{P,Q}^{M,N}[z] = \overline{H}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\phi(\xi)\, z^{\xi}\, d\xi, \qquad \ldots (5)$$

where $i = \sqrt{-1}$,

$$\phi(\xi) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)}, \qquad \ldots (6)$$

In equation (5) and (6) a_j (j = 1,…,P), b_j (j = 1,…,Q) both are complex parameters.

The following necessary condition of $\overline{H}$ function given Buschman and Srivastava [2]:

$$\Omega \equiv \sum_{j=1}^{M} \beta_j + \sum_{j=1}^{N} A_j \alpha_j - \sum_{j=M+1}^{Q} B_j \beta_j - \sum_{j=N+1}^{P} \alpha_j > 0\,, \qquad \ldots (7)$$

and $$|\arg(z)| < \frac{1}{2}\pi\,\Omega\,, \qquad \ldots (8)$$

where Ω in equation (7).

Literature Review: The H-Function- Introduced by Charles Fox the H-function is a generalized Mellin - Barnes type contour integral. It serves as a powerful mathematical tool capable of unifying almost all special functions used in physics and applied engineering mathematics, legendre, Bessel and Hyper - geometric functions.

Evolution and Extension - Papers such as those by Gupta and Shekhawat introduced combinations of H-functions and polynomials to derive more adaptable, exact solutions for shafts. Generalized Polynomials in Shaft Displacement Studies Srivastava Polynomials - The incorporation of generalized polynomials into the boundary equations allow for high levels of adaptability. These polynomial expansions provide a structural representation for varying boundary functions.

Unification: Studies have shown that when these polynomials are evaluated with products of the H - function, the resulting solutions are heavily generalized encompassing numerous special cases previously studied by researchers.

Methodology: The integral is

$$\int_0^{\lambda} \left(\cos\frac{\pi\sigma x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\sigma-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_1},\ldots,\right.$$

$$\left. y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_s}\right] \overline{H}_{P,Q}^{M,N}\left[z\left(\tan\frac{\pi x}{2\lambda}\right)^{2h}\right] dx$$

$$= \frac{\lambda 2^{2\sigma-\mu+2\sum_{i=1}^{s} k_i\alpha_i}}{\Gamma(2\sigma)\sqrt{\pi}} \sum_{\alpha_1=0}^{[N_1/M_1]} \cdots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} \cdots \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}$$

$$.B[N_1,\alpha_1;\ldots;N_s,\alpha_s] y_1^{\alpha_1} \ldots y_s^{\alpha_s} \overline{H}_{P+2,Q+1}^{M+1,N+1}\left[z\,4^h \left| \begin{matrix} (1-\sigma+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i, h; 1), \\ (\frac{1}{2}-\sigma+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i, h), \end{matrix} \right. \right.$$

$$\left. \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,p}, (\mu-\sum_{i=1}^{s} k_i\alpha_i, 2h) \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right], \qquad \ldots (10)$$

where $k_i > 0 (i=1,\ldots,s), h > 0, 2\sigma > Re\left(\mu - 2k\frac{b_j}{\beta_j}\right) > 0$ M is non- negative positive integer and coefficient B $(N_1,\alpha_1;\ldots;N_s,\alpha_s]$ are constants and real/complex.

Evaluation of Equation (10): In equation (10) integral can be established, use of the $\overline{H}$-function in equation (5) and definition of polynomial given by equation (4). We get above result.

Solution of problem: Here solution of problem to be established:

$$\psi(x,t) = \frac{1}{2^{\mu}\sqrt{\pi}} \sum_{\alpha_1=0}^{[N_1/M_1]} \cdots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} \cdots \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}$$

$$.B(N_1,\alpha_1;\ldots;N_s,\alpha_s] y_1^{\alpha_1} \ldots y_s^{\alpha_s} \frac{2^{2\tau+2\sum_{i=1}^{s} k_i\alpha_i+1}}{\Gamma(2\tau)} H_{P+2,Q+1}^{M+1,N+1}$$

$$.\left[z\,4^{h} \left| \begin{matrix} (1-\tau+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i, h;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},(\mu-\sum_{i=1}^{s} k_i\alpha_i,2h) \\ (\frac{1}{2}-\tau+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i,h),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] \left(\cos\frac{\pi\tau x}{\lambda}\right)\left(\cos\frac{\pi\tau Rt}{\lambda}\right), \qquad \ldots (11)$$

which holds true under conditions for (10).

Derivation on (11): Solution of problem written as ([6], p.125 (4)].

$$\psi(x,t) = \frac{1}{2}a_0 + \sum_{\tau=1}^{\infty} a_\tau \left(\cos\frac{\pi\tau x}{\lambda}\right)\left(\cos\frac{\pi\tau Rt}{\lambda}\right), \qquad \ldots (12)$$

When t = 0, equation (3), we get

$$\left(\sin\frac{\pi x}{2\lambda}\right)^{2\sigma-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_1},\ldots, y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_s} \right]$$

$$\overline{H}_{P,Q}^{M,N}\left[z\left(\tan\frac{\pi x}{2\lambda}\right)^{2h} \right] = \frac{1}{2}a_0 + \sum_{\tau=1}^{\infty} a_\tau \left(\cos\frac{\pi x}{\lambda}\right), \qquad \ldots (13)$$

Multiplying both the sides of (13) by $\left(\cos\frac{\pi\sigma x}{\lambda}\right)$ after that integrating w.r.t. x from 0 to λ, we have

$$\int_0^{\lambda} \left(\cos\frac{\pi\sigma x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\sigma-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}$$

$$\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_1},\ldots, y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_s} \right]$$

$$.\overline{H}_{P,Q}^{M,N}\left[z\left(\tan\frac{\pi x}{2\lambda}\right)^{2h} \right] = \frac{1}{2}a_0 \int_0^{\lambda} \left(\cos\frac{\pi\sigma x}{\lambda}\right)dx$$

$$+\sum_{\tau=1}^{\infty} a_{\tau} \int_0^{\lambda} \left(\cos\frac{\pi\tau x}{\lambda}\right)\left(\cos\frac{\pi\sigma x}{\lambda}\right) dx\,, \qquad \ldots (14)$$

Using equation (10), we get

$$a_{\tau} = \sum_{\alpha_1=0}^{[N_s/M_s]} \cdots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} \cdots \frac{(-N_s)_{M_sN_s}}{\alpha_s!} B[N_1,\alpha_1;\ldots;N_s,\alpha_s]\; y_1^{\alpha_1}\ldots y_s^{\alpha_s}$$

$$.\frac{2^{2\tau-\mu+\sum_{i=1}^{s} k_i\alpha_i+1}}{\Gamma(2\tau)\sqrt{\pi}} \overline{H}_{P+2,Q+1}^{M+1,N+1}\left[z\,4^h \left| \begin{matrix} (1-\tau+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i,h;1),(a_j,\alpha_j;A_j), \\ (\frac{1}{2}-\tau+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i,h),(b_j,\beta_j)_{1,M}, \end{matrix} \right. \right.$$

$$\left. \begin{matrix} (a_j,\alpha_j)_{N+1,P},(\mu-\sum_{i=1}^{s} k_i\alpha_i,2h) \\ (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right], \qquad \ldots (15)$$

Using (12), (13), we get desired result of (15).

Special Results:

(1) If M = 1, N = 3 = P = Q, z = – z, Use

$$g(\gamma,\eta,\nu,p;z) = \frac{a_{d-1}\,\Gamma(p+1)\,\Gamma(\frac{1}{2}+\frac{\nu}{2})}{(-1)^p\,2^{2+p}\sqrt{\pi}\,\Gamma(\gamma)\Gamma(\gamma-\frac{\nu}{2})}$$

$$.\overline{H}_{3,3}^{1,3}\left[-z \left| \begin{matrix} (1-\gamma,1;1),(1-\gamma+\frac{\nu}{2},1;1),(1-\eta,1;1+p) \\ (0,1),\left(-\frac{\nu}{2},1;1\right),(-\eta,1;1+p) \end{matrix} \right. \right],$$

where $a_d = \dfrac{2^{1-d}\,\pi^{-d/2}}{\Gamma(d/2)}$ [9]

(a) $\displaystyle \int_0^{\lambda} \left(\cos\frac{\pi\sigma x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\sigma-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}$

$$\left[y_1 \left(\tan \frac{\pi x}{2\lambda} \right)^{2k_1}, \ldots, y_s \left(\tan \frac{\pi x}{2\lambda} \right)^{2k_s} \right] g \left[\gamma, \eta, \nu, p ; z \left(\tan \frac{\pi x}{2\lambda} \right)^{2h} \right]$$

$$= \frac{a_{d-1} \Gamma(p+1) \Gamma(\frac{1}{2} + \frac{\nu}{2})}{(-1)^p\, 2^{2+p} \sqrt{\pi}\, \Gamma(\gamma) \Gamma(\gamma - \frac{\nu}{2})} \frac{\lambda\, 2^{2\sigma - \mu + 2\sum_{i=1}^{s} k_i \alpha_i}}{\Gamma(2\sigma)}$$

$$.\sum_{\alpha_1=0}^{[N_1/M_1]} \ldots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1\alpha_1}}{\alpha_1 !} \ldots \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s !} B[N_1, \alpha_1; \ldots; N_s, \alpha_s] y_1^{\alpha_1} \ldots y_s^{\alpha_s}$$

$$.\overline{H}_{5,4}^{2,4} \left[-z\, 4^h \left| \begin{matrix} (1-\tau+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i, h; 1), (1-\gamma,1;1), (1-\gamma+\frac{\nu}{2},1;1), (1-\eta,1;1+p), (\mu-\sum_{i=1}^{s} k_i\alpha_i, 2h) \\ (\frac{1}{2}-\tau+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i, h), (0,1), (-\frac{\nu}{2},1;1), (-\eta,1;1+p) \end{matrix} \right. \right] \quad \ldots (16)$$

valid conditions as required for equation (10). From equation (13), we get

$$\text{(b)} \qquad \psi(x,t) = \frac{1}{2^{\mu}\sqrt{\pi}} \sum_{\alpha_1=0}^{[N_1/M_1]} \ldots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1\alpha_1}}{\alpha_1 !} \ldots \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s !}$$

$$.B[N_1, \alpha_1; \ldots; N_s, \alpha_s] y_1^{\alpha_1} \ldots y_s^{\alpha_s} \frac{a_{d-1} \Gamma(p+1) \Gamma(\frac{1}{2} + \frac{\nu}{2})}{(-1)^p\, 2^{2+p}\, \Gamma(\gamma) \Gamma(\gamma - \frac{\nu}{2})}$$

$$.\frac{2^{2\tau + 2\sum_{i=1}^{s} k_i\alpha_i + 1}}{\Gamma(2\tau)} \overline{H}_{5,4}^{2,4} \left[-z\, 4^h \left| \begin{matrix} (1-\tau+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i, h; 1), (1-\gamma,1;1), (1-\gamma+\frac{\nu}{2},1;1), (1-\eta,1;1+p), (\mu-\sum_{i=1}^{s} k_i\alpha_i, 2h) \\ (\frac{1}{2}-\tau+\frac{\mu}{2}-\sum_{i=1}^{s} k_i\alpha_i, h), (0,1), (-\frac{\nu}{2};1), (-\eta,1;1+p) \end{matrix} \right. \right] \cos\left(\frac{\pi\tau x}{\lambda}\right) \left(\cos \frac{\pi\tau R t}{\lambda} \right), \quad \ldots (17)$$

valid under the conditions as required equation (13).

(2) If M = 1, N = 3 = P and Q = 2, replacing $z = -(1+\varepsilon)^{-2}$ and the using

$$\beta F(d;\varepsilon) = -\frac{1}{4\pi^{1/2}(1+\varepsilon)^2}\overline{H}^{1,3}_{3,2}\left[-(1+\varepsilon)^{-2}\left|\begin{matrix}(0,1;1),(0,1;1),(-1/2,1;d)\\(0,1),(-1,1;1+d)\end{matrix}\right.\right], [9,\text{p.}4121,\text{eq.}(5)]$$

We obtain equation (11)

(a) $$\int_0^{\lambda}\left(\cos\frac{\pi\sigma x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\sigma-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S^{M_1,\ldots,M_s}_{N_1,\ldots,N_s}$$

$$\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_1},\ldots,y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2k_s}\right]\beta F(d;\varepsilon)\,dx$$

$$= -\frac{\lambda e^{2\sigma-\mu+2\sum_{i=1}^{s}k_i\alpha_i-2}}{\Gamma(2\sigma)\sqrt{\pi}^{(d+1)/2}(1+\varepsilon)^2}\sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!}\cdots\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}$$

$$.\ B[N_1,\alpha_1;\ldots;N_s,\alpha_s)\,y_1^{\alpha_1}\ldots y_s^{\alpha_s}$$

$$.\overline{H}^{2,4}_{5,3}\left[-(1+\varepsilon)^{-2}\,4^h\left|\begin{matrix}(1-\sigma+\frac{\mu}{2}-\sum_{i=1}^{s}k_i\alpha_i,h;1),(0,1;1),(0,1;1),(-1/2,1;d),(\mu-\sum_{i=1}^{s}k_i\alpha_i,2h)\\(\frac{1}{2}-\sigma+\frac{\mu}{2}-\sum_{i=1}^{s}k_i\alpha_i,h),(0,1),(-1,1;1+d)\end{matrix}\right.\right] \ldots (18)$$

valid under conditions as required equation (11). From equation (13), We have

(b) $$\psi(x,t) = -\frac{1}{2^{\mu+1}\sqrt{\pi}^{(d+1)/2}(1+\varepsilon)^2}\sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!}\cdots\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}$$

$$.B[N_1,\alpha_1;\ldots;N_s,\alpha_s]\,y_1^{\alpha_1}\ldots y_s^{\alpha_s}\,\frac{2^{2\tau+2\sum_{i=1}^{s}k_i\alpha_i}}{\Gamma(2\tau)}$$

$$.\overline{H}^{2,4}_{5,3}\left[-(1+\varepsilon)^{-2}\,4^h\left|\begin{matrix}(1-\tau+\frac{\mu}{2}-\sum_{i=1}^{s}k_i\alpha_i,h;1),(0,1;1),(0,1;1),(-1/2,1;d),(\mu-\sum_{i=1}^{s}k_i\alpha_i,2h)\\(\frac{1}{2}-\tau+\frac{\mu}{2}-\sum_{i=1}^{s}k_i\alpha_i,h),(0,1),(-1,1;1+d)\end{matrix}\right.\right]\cos\left(\frac{\pi\tau x}{\lambda}\right)\left(\cos\frac{\pi\tau Rt}{\lambda}\right), \ldots (19)$$

valid conditions as required for equation (12).

(3) (a) If $N_{i'} \to 0 (i' = 1,...,s)$, Equation (6.1) is in following the form

$$\int_0^{\lambda} \left(\cos\frac{\pi\sigma x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\sigma-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} g\left[\gamma,\eta,\nu,p\,;z)\left(\tan\frac{\pi x}{2\lambda}\right)^{2h}\right]$$

$$= \frac{a_{d-1}\Gamma(p+1)\,\Gamma(\frac{1}{2}+\frac{\nu}{2})}{(-1)^p\,2^{2+p}\sqrt{\pi}\,\Gamma(\gamma)\Gamma(\gamma-\frac{\nu}{2})}\,\frac{\lambda\,2^{2\sigma-\mu}}{\Gamma(2\sigma)}$$

$$.\overline{H}^{2,4}_{5,4}\left[-z\,4^h \left| \begin{matrix} (1-\tau+\frac{\mu}{2},h\,;1),(1-\gamma,1;1),(1-\gamma+\frac{\nu}{2},1;1),(1-\eta,1;1+p),(\mu,2h) \\ (\frac{1}{2}-\tau+\frac{\mu}{2},h)\,,(0,1),(-\frac{\nu}{2},1;1),(-\eta,1;1+p) \end{matrix} \right.\right] \quad \ldots (20)$$

valid conditions as required for equation (18).

(b) If $N_{i'} \to 0 (i' = 1,...,s)$, Equation (6.2) is in the following form

$$\psi(x,t) = \frac{1}{2^{\mu}\sqrt{\pi}}\,\frac{a_{d-1}\Gamma(p+1)\,\Gamma(\frac{1}{2}+\frac{\nu}{2})}{(-1)^p\,2^{2+p}\,\Gamma(\gamma)\Gamma(\gamma+\frac{\nu}{2})}\,\frac{2^{2\tau+1}}{\Gamma(2\tau)}$$

$$.\overline{H}^{2,4}_{5,4}\left[-z\,4^h \left| \begin{matrix} (1-\tau+\frac{\mu}{2},h\,;1),(1-\gamma,1;1),(1-\gamma+\frac{\nu}{2},1;1),(1-\eta,1;1+p),(\mu,2h) \\ (\frac{1}{2}-\tau+\frac{\mu}{2},h)\,,(0,1),(-\frac{\nu}{2},1;1),(-\eta,1;1+p) \end{matrix} \right.\right].\cos\left(\frac{\pi\tau x}{\lambda}\right)\left(\cos\frac{\pi\tau\, Rt}{\lambda}\right),$$

$$\ldots (21)$$

valid the conditions as required for equation (19).

4. Letting s = 2 in equation (11) and (13), we get result obtained by Chaurasia and Lata [5].

5. If $A_j = 1 = B_j$, s = 2, $K \to 0$ in equation (11) and (13), we get useful result found by Chaurasia and Godika [3].

Conclusion: The analytical models developed are highly valuable for understanding and predicting rotational dynamics and twist mechanics in engineering applications. The use of special functions and their applications is becoming increasingly important technical applications space research and nuclear reactor give rise to several problems of the application of special functions.

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